

# Online Appendix

## Appendix A Proofs

### A.1 Derivation of equation (4)

The government's problem is to choose  $B' \in (-\infty, \bar{B}]$  to maximize the expected end-of-period value, where expectation is over the realization of  $\epsilon$ :

$$\begin{aligned} V_1(s) &= \max_{B'} \left\{ F(\sigma^{-1}\Delta(s, B'))V^R(s, B') + \int_{\sigma^{-1}\Delta(s, B')}^1 [V^D(s, B') + \sigma\epsilon] dF(\epsilon) \right\} \\ &= \max_{B'} \left\{ V^D(s, B') + F(\sigma^{-1}\Delta(s, B'))\Delta(s, B') + \sigma \int_{\sigma^{-1}\Delta(s, B')}^1 \epsilon dF(\epsilon) \right\}. \end{aligned}$$

Using integration by parts, we can re-write this as (4).

### A.2 Proof of Proposition 1

*Proof.* Let us define  $H : [0, 1] \rightarrow \mathbb{R}$  by

$$\begin{aligned} H(\epsilon) &\equiv u(Y - B + F(\epsilon)q_{EG}(B')B') - u(Y + F(\epsilon)q_{EG}(B')B') - \sigma\epsilon \\ &\quad + \beta\mathbb{E}[V(s')|B'] - \beta\underline{V}^D. \end{aligned} \tag{17}$$

Equation (13) is satisfied at  $\tilde{\epsilon}$  if and only if  $H(\tilde{\epsilon}) = 0$ . Note that  $\epsilon$  only enters  $H$  via the functions  $u$  and  $F$ , both of which are continuous. Hence,  $H$  is continuous. Premise (i) states that  $H(1) \geq 0$ . Premise (ii) states that  $H(0) \leq 0$ . By continuity there is at least one  $\epsilon \in [0, 1]$  such that  $H(\epsilon) = 0$ . If  $H(1) > 0$  and  $H(0) < 0$ , then the  $\epsilon$  that solves  $H(\epsilon) = 0$  must be strictly interior.  $\square$

### A.3 Proof of Proposition 2

*Proof.* For a given  $(Y, B, B')$ , let  $q_{EG}$  denote  $q_{EG}(B')$ . If  $q_{EG} = 0$ , the only equilibrium price is zero. If  $B' \leq 0$ , the only equilibrium price is  $q_{EG} = R^{-1}$ , as assets always trade at the risk-free price. Thus the proposition is trivially true for  $B' \leq 0$  and  $q_{EG} = 0$ . Henceforward, assume  $q_{EG}B' > 0$ .

Define the function  $g : [0, 1] \rightarrow \mathbb{R}$  by:

$$g(\epsilon) \equiv u(Y - B + F(\epsilon)q_{EG}B') - u(Y + F(\epsilon)q_{EG}B') + \beta\mathbb{E}[V(s')|B'] - \beta\underline{V}^D. \tag{18}$$

Then  $H : [0, 1] \rightarrow \mathbb{R}$  defined in equation (17) in the proof of Proposition 1 can be written as:

$$H(\epsilon) \equiv g(\epsilon) - \sigma\epsilon. \tag{19}$$

Zero is an equilibrium if and only if  $H(0) \leq 0$ ;  $q_{EG}$  is an equilibrium price if and only if  $H(1) \geq 0$ ; and  $\tilde{\epsilon}$  solve equation (13) if and only if  $H(\tilde{\epsilon}) = 0$ . We consider the cases of  $B < 0$ ,  $B = 0$ , and  $B > 0$  in turn:

**If  $B < 0$ :** If  $B < 0$ , then  $g(\epsilon) > 0$  for all  $\epsilon \in [0, 1]$ . This uses the fact that  $\beta\mathbb{E}[V(s')|B'] - \beta\underline{V}^D \geq 0$ . This inequality follows from the fact that  $V(s) \geq \max_{B', \leq \bar{B}} \mathbb{E}[V^D(s, B') + \sigma\epsilon] \geq \underline{V}^D$ . As  $g$  is continuous on the compact domain  $[0, 1]$ , it achieves a minimum  $\underline{g} \equiv \min_{\epsilon \in [0, 1]} g(\epsilon) > 0$ . If  $\sigma < \underline{g}$ , then  $H(\epsilon) = g(\epsilon) - \sigma\epsilon \geq \underline{g} - \sigma\epsilon > \sigma(1 - \epsilon) \geq 0$ . Hence,  $H(\epsilon) > 0$  for all  $\epsilon$ , and  $q_{EG}(B')$  is the only possible equilibrium price. Thus, the proposition holds for  $B < 0$  by setting  $K = \underline{g} > 0$ .

**If  $B = 0$ :** If  $B = 0$ , then  $g(\epsilon) = \beta\mathbb{E}[V(s')|B'] - \beta\underline{V}^D$ , which is independent of  $\epsilon$ . If  $g = 0$ , then  $H(\epsilon) = -\sigma\epsilon$ , and the only possible equilibrium price is 0 for any  $\sigma > 0$ . If  $g > 0$ , then, letting  $K = g > 0$ , for  $\sigma < K$ , we have  $H(\epsilon) = K - \sigma\epsilon > 0$ . In this case,  $H(\epsilon) \geq H(1) > 0$  and the only possible price is

$q_{EG}$ . Thus, the proposition holds for  $B = 0$ .

**If  $B > 0$ :** If  $B > 0$ , then concavity of  $u$  implies that  $g(\epsilon)$  is strictly increasing in  $\epsilon$ . In particular,

$$g'(\epsilon) = [u'(Y - B + F(\epsilon)q_{EG}B') - u'(Y + F(\epsilon)q_{EG}B')] F'(\epsilon)q_{EG}B' > 0,$$

where the last inequality uses the fact that  $F'(\epsilon) \geq \alpha > 0$  and  $q_{EG}B' > 0$ . Let

$$K \equiv \min_{\epsilon \in [0,1]} [u'(Y - B + F(\epsilon)q_{EG}B') - u'(Y + F(\epsilon)q_{EG}B')] \underline{f}q_{EG}B' > 0,$$

where the minimum exists as  $u'$  is a continuous function and is strictly positive given that  $g'(\epsilon) > 0$  for all  $\epsilon \in [0,1]$ . Then  $H'(\epsilon) = g'(\epsilon) - \sigma \geq K - \sigma$ . If  $\sigma < K$ , then  $H'(\epsilon) > 0$ . Hence there is at most one  $\tilde{\epsilon}$  such that  $H(\tilde{\epsilon}) = 0$ . If  $H(0) \leq 0 \leq H(1)$ , then  $\{0, q_{EG}, F(\tilde{\epsilon})q_{EG}\}$  are all equilibrium prices. If  $H(1) > H(0) > 0$ , then only  $q_{EG}$  is an equilibrium price. If  $H(0) < H(1) < 0$ , then only zero is an equilibrium price. Thus, the proposition holds for  $B > 0$ .  $\square$

## A.4 Proof of Proposition 3

*Proof.* Using  $H$  defined by (17) in the proof of Proposition 1, we have  $\tilde{q} = 0$  satisfies (8) if  $H(0) \leq 0$ ;  $\tilde{q} = q_{EG}(B')$  if  $H(1) \geq 0$ ; and  $\tilde{q} \in (0, q_{EG}(B'))$  if  $H(\tilde{\epsilon}) = 0$  for some  $\tilde{\epsilon} \in (0, 1)$ . Define

$$h(x) \equiv u(Y - B + x) - u(Y + x). \quad (20)$$

We have

$$H'(\epsilon) = h'(F(\epsilon)q_{EG}(B')B') F'(\epsilon)q_{EG}(B')B' - \sigma. \quad (21)$$

By definition of  $h$ ,

$$\begin{aligned} h'(x) &= u'(Y - B + x) - u'(Y + x) \\ h''(x) &= u''(Y - B + x) - u''(Y + x). \end{aligned}$$

As  $B > 0$  and  $u$  strictly concave, we have  $h'(x) > 0$  for  $x \geq 0$ . If  $u''$  is strictly increasing, then  $h''(x) < 0$ . Hence, as  $\epsilon$  increases,  $h'(F(\epsilon)q_{EG}(B')B')$  strictly decreases and  $F'(\epsilon)$  weakly decreases. As both are positive, their product decreases and  $H'$  is strictly decreasing. This establishes that there are at most two roots to  $H$ ; that is, there are at most two interior prices that satisfy (13). If  $H(0) \leq 0$ , then  $\tilde{q} = 0$  satisfies (8), and there are at most three possible equilibrium prices, two of which being interior. If  $H(0) > 0$ , then  $\tilde{q} = 0$  is not an equilibrium. As  $H(0) > 0$  and  $H'$  strictly decreasing, there is at most one interior solution  $\tilde{\epsilon}$  to (13), with  $H'(\tilde{\epsilon}) < 0$ . If there is such a  $\tilde{\epsilon} < 1$ , then  $H(1) < 0$  and  $\tilde{q} = F(\tilde{\epsilon})q_{EG}(B')$  is the only equilibrium price. Otherwise,  $H(\epsilon) \geq 0$  for all  $\epsilon \in [0, 1]$ , and  $\tilde{q} = q_{EG}(B')$  is the only equilibrium price.  $\square$

## A.5 Proof of Proposition 4

*Proof.* We first establish that  $\mathbb{E}[V(s')|B'] \geq \underline{V}^D + \sigma\mathbb{E}\epsilon$  for all  $B'$ . To see this, it is always feasible to issue zero new debt and default with probability one on maturing debt:

$$\begin{aligned} V(s) &\geq u(Y) + \sigma\mathbb{E}\epsilon + \beta\underline{V}^D \\ &= \sigma\mathbb{E}\epsilon + u(Y) - \mathbb{E}u(Y') + \underline{V}^D, \end{aligned}$$

where the second line uses the fact that  $\underline{V}^D \equiv \mathbb{E}u(Y')/(1-\beta)$ . Taking expectation over  $s'$  for any  $B' \leq \bar{B}$ , we have

$$\mathbb{E}[V(s')|B'] \geq \underline{V}^D + \sigma\mathbb{E}\epsilon. \quad (22)$$

From the definitions of  $V_{CK}^R$  and  $V_{CK}^D$ , we can re-write  $\mathbb{B}_{CK}$  as:

$$\mathbb{B}_{CK}(Y, B) \equiv \{B' \in [0, \bar{B}] \mid u(Y - B) - u(Y) + \beta \mathbb{E}[V(s')|B'] - \beta \underline{V}^D \leq 0\}. \quad (23)$$

Equation (22) implies that if  $B' \in \mathbb{B}_{CK}(Y, B)$ , then

$$\begin{aligned} 0 &\geq u(Y - B) - u(Y) + \beta \mathbb{E}[V(s')|B'] - \beta \underline{V}^D \\ &\geq u(Y - B) - u(Y) + \beta \sigma \mathbb{E}\epsilon. \end{aligned}$$

That is, (14) is a necessary condition for  $\mathbb{B}_{CK} \neq \emptyset$ . To show that (14) is also a sufficient condition, suppose (14) satisfied. For  $B'$  larger than the natural borrowing limit (which we have assumed is strictly less than  $\bar{B}$ ), repayment is infeasible for any endowment realization and default occurs with probability one. Hence, there exists a  $\tilde{B} \in (0, \bar{B})$  such that  $\mathbb{E}[V(s')|B' \geq \tilde{B}] = \sigma \mathbb{E}\epsilon + \underline{V}^D$ . Therefore,  $[\tilde{B}, \bar{B}] \subset \mathbb{B}_{CK}(Y, B)$ . As  $[\tilde{B}, \bar{B}] \neq \emptyset$  when (14) holds,  $\mathbb{B}_{CK}$  is not empty.  $\square$

## A.6 Proof of Proposition 5

*Proof.* If  $q(s, B'_i)$  is interior for  $i = 1, 2$ , there exist respective  $\tilde{\epsilon}_i$  that satisfy (13). As  $q(s, B'_i)/q_{EG}(B'_i) = F(\tilde{\epsilon}_i)$ , and  $F$  is strictly increasing on its support, we have  $\tilde{\epsilon}_1 \leq \tilde{\epsilon}_2$ . By (13), we also have:

$$V^R(s, B'_i) = V^D(s, B'_i) + \sigma \tilde{\epsilon}_i,$$

for  $i = 1, 2$ . Using this, the expected payoff from  $B'_i$  at the time of auction can be written:

$$\begin{aligned} &\mathbb{E} \max \{V^R(s, B'_i), V^D(s, B'_i) + \sigma \tilde{\epsilon}\} \\ &= \mathbb{E} \max \{V^D(s, B'_i) + \sigma \tilde{\epsilon}_i, V^D(s, B'_i) + \sigma \epsilon\} \\ &= V^D(s, B'_i) + \sigma \mathbb{E} \max \{\tilde{\epsilon}_i, \epsilon\}. \end{aligned}$$

As  $q(s, B'_1)B'_1 \leq q(s, B'_2)B'_2$ , we have  $V^D(s, B'_1) \leq V^D(s, B'_2)$ . This, plus the fact that  $\tilde{\epsilon}_1 \leq \tilde{\epsilon}_2$  implies that  $B'_2$  weakly dominates  $B'_1$  as a debt choice. If either  $q(s, B'_1)B'_1 < q(s, B'_2)B'_2$  or  $\tilde{\epsilon}_1 < \tilde{\epsilon}_2$ , the preference for  $B'_2$  is strict.  $\square$

## A.7 Proof of Proposition 6

*Proof.* Fix  $s$  and  $B' < (1 - \lambda)B$  and let  $q_{EG}$  denote  $q_{EG}(s, B')$ . For  $\tilde{q} \in [0, q_{EG}]$ , define  $\hat{F}(\tilde{q})$  by:

$$\hat{F}(\tilde{q}) \equiv F\left(\frac{1}{\sigma} \left[ u(Y - (r^* + \lambda)B + \tilde{q} \times [B' - (1 - \lambda)B]) - u(Y + \tilde{q} \times [B' - (1 - \lambda)B]) + \beta \mathbb{E}[V(s')|s, B'] - \beta \mathbb{E}V^D(s') \right] \right).$$

A  $\tilde{q} \in [0, q_{EG}]$  is an equilibrium price if and only if  $\tilde{q}/q_{EG} = \hat{F}(\tilde{q})$ . Note that  $\hat{F}$  maps  $[0, q_{EG}]$  into  $[0, 1]$ , is weakly decreasing (as  $B' < (1 - \lambda)B$ ), and continuous. Hence, there exists one and only one  $\tilde{q}$  that satisfies the equilibrium condition.  $\square$

## A.8 Proof of Proposition 7

*Proof.* Since  $q(s, B'_i)$  are interior equilibrium prices for  $i = 1, 2$ , there exist thresholds  $\tilde{\epsilon}_i \in [0, 1]$  such that:

$$V^R(s, B'_i) = V^D(s, B'_i) + \sigma \tilde{\epsilon}_i, \text{ for } i = 1, 2.$$

Conditional on auctioning  $B'_i$ , the expected payoff is:

$$\begin{aligned} & \int_0^1 \max \{V^R(s, B'_i), V^D(s, B'_i) + \sigma\epsilon\} dF(\epsilon) \\ &= \int_0^1 \max \{V^D(s, B'_i) + \sigma\tilde{\epsilon}_i, V^D(s, B'_i) + \sigma\epsilon\} dF(\epsilon) \\ &= V^D(s, B'_i) + \sigma \int_0^1 \max \{\tilde{\epsilon}_i, \epsilon\} dF(\epsilon). \end{aligned}$$

From condition (i) in the proposition statement, we have  $\tilde{\epsilon}_1 \geq \tilde{\epsilon}_2$ , with strict inequality if (i) is strict. As the default value is increasing in net auction revenue, from premise (ii) in the proposition, we have  $V^D(s, B'_1) \geq V^D(s, B'_2)$ , with strict inequality if (ii) is strict. Thus, the expected value from auctioning  $B'_1$  is weakly greater than  $B'_2$ , and strictly if either (i) or (ii) is strict.  $\square$

*Proof.* Since  $q(s, B'_i)$  are interior equilibrium prices for  $i = 1, 2$ , there exist thresholds  $\tilde{\epsilon}_i \in [0, 1]$  such that:

$$V^R(s, B'_i) = V^D(s, B'_i) + \sigma\tilde{\epsilon}_i, \text{ for } i = 1, 2.$$

Conditional on auctioning  $B'_i$ , the expected payoff is:

$$\begin{aligned} & \int_0^1 \max \{V^R(s, B'_i), V^D(s, B'_i) + \sigma\epsilon\} dF(\epsilon) \\ &= \int_0^1 \max \{V^D(s, B'_i) + \sigma\tilde{\epsilon}_i, V^D(s, B'_i) + \sigma\epsilon\} dF(\epsilon) \\ &= V^D(s, B'_i) + \sigma \int_0^1 \max \{\tilde{\epsilon}_i, \epsilon\} dF(\epsilon). \end{aligned}$$

From condition (i) in the proposition statement, we have  $\tilde{\epsilon}_1 \geq \tilde{\epsilon}_2$ , with strict inequality if (i) is strict. As the default value is increasing in net auction revenue, from premise (ii) in the proposition, we have  $V^D(s, B'_1) \geq V^D(s, B'_2)$ , with strict inequality if (ii) is strict. Thus, the expected value from auctioning  $B'_1$  is weakly greater than  $B'_2$ , and strictly if either (i) or (ii) is strict.  $\square$

## Appendix B Mixed Strategy Equilibria: $\sigma = 0$

In this appendix, we formalize the notion that as  $\sigma \rightarrow 0$ , the equilibrium converges to a mixed-strategy equilibrium in which the government randomizes over default or repayment. That is, suppose the government's decision at settlement is to pick a probability of repayment:  $p \in [0, 1]$ . When facing a price  $\tilde{q}$ , the government's best response is:

$$\begin{aligned} p &= 1 \text{ if } u(Y - B + \tilde{q}B') + \beta\mathbb{E}[V(s')|B'] > u(Y + \tilde{q}B') + \beta\underline{V}^D; \\ p &= 0 \text{ if } u(Y - B + \tilde{q}B') + \beta\mathbb{E}[V(s')|B'] < u(Y + \tilde{q}B') + \beta\underline{V}^D; \text{ and} \\ p &\in [0, 1] \text{ if } u(Y - B + \tilde{q}B') + \beta\mathbb{E}[V(s')|B'] = u(Y + \tilde{q}B') + \beta\underline{V}^D. \end{aligned} \tag{24}$$

Similarly, the lenders' best response to an anticipated  $p$  at settlement is to bid  $\tilde{q} = p \times q_{EG}$  at auction. A mixed-strategy equilibrium price is a pair  $(p, \tilde{q})$  that satisfies  $\tilde{q} = pq_{EG}$  and equation (24). Following the steps behind Proposition 1, it is straightforward to see that if  $(p = 1, \tilde{q} = q_{EG})$  satisfies the first inequality in (24), and  $(p = 0, \tilde{q} = 0)$  satisfies the second, then there is also a unique  $p \in (0, 1)$  with an interior  $\tilde{q} = pq_{EG} \in (0, q_{EG})$  that satisfies the third line of (24).

Reminiscent of Harsanyi (1973) purification, the mixed strategy price is the limit of the pure-

strategy interior price as  $\sigma \rightarrow 0$ :

**Proposition A.1.** *Given  $(Y, B, B')$  with  $B' > 0$ , suppose  $V_{EG}^R(Y, B, B') > V_{EG}^D(Y, B')$  and  $V_{CK}^R(Y, B, B') < V_{CK}^D(Y)$ . Let  $\sigma_n$  be a monotone decreasing sequence converging to zero. Then there exists an integer  $N < \infty$  and a sequence  $\tilde{\epsilon}_n$  satisfying (13) for each  $\sigma_n$  for  $n > N$ . Moreover,  $p = \lim_{n \rightarrow \infty} F(\tilde{\epsilon}_n)$  and  $\tilde{q} = \lim_{n \rightarrow \infty} F(\tilde{\epsilon}_n)q_{EG}(B')$  exist and satisfy (24).*

*Proof.* First, note that  $B \neq 0$ . To see this, when  $B = 0$ , we have

$$V_{EG}^R(Y, 0, B') - V_{EG}^D(Y) = \beta (\mathbb{E}V(s') - \underline{V}^D) = V_{CK}^R(Y, 0, B') - V_{CK}^D(Y),$$

which is inconsistent with the inequalities in the proposition's premise. Hence, we take  $B' \neq 0$  in what follows. As  $(Y, B, B')$  is fixed throughout, we drop these arguments from the notation below. The premise implies  $V_{EG}^R > V_{CK}^R$ , and hence it must be the case that  $q_{EG} > 0$ .

Define  $h : [0, 1] \rightarrow \mathbb{R}$  by:

$$h(x) \equiv u(Y - B + xq_{EG}B') - u(Y + xq_{EG}B').$$

$h$  represents the net current period flow utility from repayment over default when the price is  $xq_{EG}$ . By strict concavity of  $u$  and  $q_{EG}B' > 0$ , we have  $h'(x) \geq 0$  when  $B \geq 0$ . The inequalities in the premise imply:

$$h(1) + \beta (\mathbb{E}V(s') - \underline{V}^D) > 0 > h(0) + \beta (\mathbb{E}V(s') - \underline{V}^D).$$

By continuity of  $h$ , there exists a  $p \in [0, 1]$  such that  $h(p) + \beta (\mathbb{E}V(s') - \underline{V}^D) = 0$ . At price  $\tilde{q} = pq_{EG}$ , the government is indifferent to defaulting or repaying when  $\sigma = 0$ . At price  $\tilde{q}$  and if the government randomizes by defaulting with probability  $1 - p$ , the lenders break even. Hence,  $(\tilde{q}, p)$  is a mixed strategy equilibrium.

Let  $K > 0$  be defined as in Proposition 2. Let  $\bar{\sigma} \equiv V_{EG}^R - V_{EG}^D$ , which is strictly positive by the proposition's premise. Let  $N$  be defined by:

$$N \equiv \inf\{n \geq 0 \mid \sigma_n < \min\{\bar{\sigma}, K\}\}.$$

As the sequence  $\sigma_n$  is monotonically decreasing, for  $n > N$ , we have

$$V_{EG}^R - V_{EG}^D - \sigma_n > V_{EG}^R - V_{EG}^D - \bar{\sigma} = 0,$$

and hence  $q_{EG}$  is supportable as an equilibrium price. As  $V_{CK}^R < V_{CK}^D$ , zero is also an equilibrium price. For each  $n > N$  and associated  $\sigma_n$ , by Proposition 2, there exists a unique  $\epsilon_n$  such that equation (13) holds. Define  $p_n \equiv F(\epsilon_n)$ . From (13) and the fact that  $\tilde{\epsilon}_n \leq 1$ , we have

$$0 \leq h(p_n) - h(p) = \sigma_n \tilde{\epsilon}_n \leq \sigma_n.$$

Hence, as  $\sigma_n \rightarrow 0$ ,  $h(p_n) - h(p) \rightarrow 0$ . To establish convergence in the arguments of  $h$ , define

$$\kappa \equiv \min_{x \in [0, 1]} |h'(x)|.$$

As  $h'$  is continuous and  $h'$  is either strictly positive or negative for all  $x \in [0, 1]$  given  $B \neq 0$ ,  $\kappa > 0$  is well defined. Note that by definition

$$h(p_n) - h(p) \geq \kappa |p_n - p|.$$

Hence,  $h(p_n) - h(p) \rightarrow 0$  implies  $|p_n - p| \rightarrow 0$ . □

## Appendix C Further Numerical Experiments

Several additional numerical experiments to further examine the implications of our model are reported here.

### C.1 Italy Calibration

Using real GDP from 1960Q1-2020Q3,<sup>32</sup> we estimate the same process for endowment as reported for Mexico in the text. The estimated parameters are:  $\mu_g = 0.002$ ,  $\rho_g = 0.579$ ,  $\sigma_g = 0.008$ , and  $\sigma_z = 0.0097$ .

For 2011, tax revenue net of social security contributions was 474,863 million euros which was 29.0% of GDP.<sup>33</sup> The replication data provided by Bocola and DAVIS (2016) reports that 373,000 million euros of debt was scheduled to come due in the year following 2011Q4. Debt payments as a ratio of tax revenues net of social security contributions thus totaled 78.5% and the face value of total debt securities relative to tax revenue was 334% (or 97% of GDP). 2011Q4 represented a debt burden relative to tax revenue that is the 93rd percentile of the Bocola-Davis dataset. We therefore target debt payments of 78.5% of resources at the 93rd percentile of the model's ergodic distribution of debt. For the long-term model, to match the associated face value given a 3% coupon, we set  $\lambda = 0.0286$ , or a maturity of roughly three years (35 quarters).

With regard to sovereign spreads, for the period 1999Q1-2020Q4 the spread between Italian and German five-year bonds had a standard deviation of 1.05%.<sup>34</sup>

We targeted the standard deviation of the spread and the ratio of debt payments to resources at the 93rd percentile, assuming  $\beta = 0.95$ . For the one-period debt model, this requires  $d = 0.14$  and the probability of the concerned regime of 0.4% per quarter; for the long-term bond model, it requires  $d = 0.665$  and a probability of the concerned regime of 0.3% per quarter.

The model's moments are reported in Table 2. Italy has a less volatile output process, a higher level of the debt payment coming due, and a lower volatility of the spread than Mexico. When we calibrate the base model for both the one-period and long-term bond cases, we are able to hit these targets. When we only have  $\rho = O$  beliefs, the default rate and the spread volatility essentially fall to zero with one-period bonds. In the long-term bond case with  $\rho = O$  beliefs, the temptation to engage in debt dilution leads to almost as many defaults but substantially less volatility relative to the base case. Once again, the Cole-Kehoe version with one period debt and  $\rho \in \{O, P\}$  generates a lot more default in the one-period bond case, but not much volatility, and essentially no defaults and no spread volatility with long-term bonds. All of these results mirror those found for Mexico.

### C.2 Partial Retention of Auction Revenue

Beyond allowing for long-term debt, another aspect of capturing sovereign debt crises is getting investor *returns* in default right. In Cole-Kehoe, it is assumed that the government can keep all of the proceeds of an auction, while Eaton-Gersovitz effectively assume that they can keep none. Viewed in terms of a quarterly period length, with weekly auctions, both assumptions seem extreme. This leads us to also consider the intermediate case with long-term debt where the government gets to keep 1/2 of the auction proceeds in the event of default, the rest being split

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<sup>32</sup>Italian data are from from OECD.stat (<https://stats.oecd.org>).

<sup>33</sup>Social security contributions were 211,637 million euros leading to total tax revenue of 686,500 million euros.

<sup>34</sup>The German and Italian data are 5-year yields from Deutsche Bundesbank and Banca D'Italia, respectively, as reported in Haver Analytics G10 summary statistics database.

Table 2: Simulated Moments for Italy

Model	93%ile	Default	StDev( $r - r^*$ )	$\frac{\text{StDev}(\ln c)}{\text{StDev}(\ln y)}$
	DebtPmt/Rev (Quarterly)	Frequency (Annualized)		
		<u>One-Period Bonds</u>		
Base, $\rho \in \{O, C\}$	79%	0.1%	1.1%	1.11
Optimistic, $\rho \in \{O\}$	79%	0.0%	0.0%	1.11
Cole-Kehoe, $\rho \in \{O, P\}$	74%	2.1%	0.1%	1.07
		<u>Long-Term Bonds</u>		
Base, $\rho \in \{O, C\}$	79%	0.9%	1.1%	1.41
Optimistic, $\rho \in \{O\}$	80%	0.8%	0.5%	1.44
Cole-Kehoe, $\rho \in \{O, P\}$	53%	0.0%	0.0%	1.50

Table 3: Long-Term Model Moments with Partial Retention

Model	97%ile	Default	StDev( $r - r^*$ )	$\frac{\text{StDev}(\ln c)}{\text{StDev}(\ln y)}$
	DebtPmt/Rev (Quarterly)	Frequency (Annualized)		
Base, $\rho \in \{O, C\}$	66%	5.8%	2.3%	1.51
Optimistic, $\rho = O$	67%	5.3%	1.1%	1.54
Cole-Kehoe, $\rho \in \{O, P\}$	37%	0.0%	0.0%	1.41

pro-rata among the government’s creditors (old and new) under the assumption that the auction revenue is not repatriated until all outstanding claims are settled. The assumption that bondholders receive payments (if any) in proportion to the face value of their claims reflects the *pari passu* and acceleration clauses typically included in sovereign bond contracts.

When we calibrate the model to our two Mexican moments, with maturity set to be same as in the long-term case and with the retention set to 1/2, we find that  $d = 0.455$  and concerned probability of 0.25% quarterly. The results are reported in Table 3 and, strikingly, closely mirror the results (for the 100% retention case) reported in the bottom panel of Table 1.

### C.3 Larger Interim Shocks

To examine the sensitivity of our results a larger volatility of the interim shock  $\sigma$ , we scale the shock by a factor of 100 and recalculated the results at the original calibration for Mexico. The results are virtually unchanged relative to our baseline for Mexico.

### C.4 Alternate Timing Assumption: Eaton-Gersovitz

In this section, we explore model predictions for the Eaton-Gersovitz timing, namely, the assumption that new debt can be issued only if old debt is paid off. This assumption effectively reverses the timing of the default decision and the auction. Since the shock to the default payoff

Table 4: Base Model Moments with Larger Interim Shocks

Model	97 <sup>th</sup> ile	Default	StDev( $r - r^*$ ) (Annualized)	$\frac{\text{StDev}(\ln c)}{\text{StDev}(\ln y)}$
	DebtPmt/Rev (Quarterly)	Frequency (Annualized)		
Short-Term	64%	0.4%	2.3%	1.16
Long-Term	66%	5.3%	2.2%	1.51

comes before the auction it is no longer an intraperiod shock, and given its small variance, it can be safely removed for the purpose of this exercise.

Table 5: Base Model Moments Under EG Timing

Model	97 <sup>th</sup> ile	Default	StDev( $r - r^*$ ) (Annualized)	$\frac{\text{StDev}(\ln c)}{\text{StDev}(\ln y)}$
	DebtPmt/Rev (Quarterly)	Frequency (Annualized)		
Short-Term	254%	0.3%	0.1%	1.78
Long-Term	77%	2.1%	0.3%	1.22

The results, displayed in Table 5, show that in both the one-period and long-term debt cases, debt payments (and debt levels) rise substantially; in the one-period case, it rises by nearly an order of magnitude. The expansion in debt speaks to the importance of the commitment to use auction revenue for repayment of inherited debt implicit in the EG timing. But, aside from this, both default frequency and spread volatility fall to very low levels in the one-period debt case. While long-term debt case generates a higher frequency of default, the volatility of spreads still remains quite low. These findings are consistent with what the literature finds if nonlinear default costs are ignored. Also, as shown in Aguiar et al. (2016), even allowing for the possibility of nonlinear default costs cannot deliver the observed spread volatility for Mexico.